DEUTERON FORMATION IN HEAVY ION COLLISIONS WITHIN THE FADDEEV APPROACH

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Abstract

We address the formation of correlations in a general nonequilibrium situation. As an example we calculate the formation of light composite particles in a heavy ion collision. In particular we study the formation of deuterons via three-body reactions in some detail. To calculate the relevant reaction rates we solve the Faddeev equation which is consistently modified to include the self energy corrections and Pauli blocking factors due to surrounding nucleons. We find that the time scales of the reaction as well as the number of emitted deuterons change as medium dependent rates are used in the calculation.

1 Introduction

The description of the dynamics of an interacting many-body system is particularly difficult when the quasiparticle approach reaches its limits. That may be the case when the residual interaction is strong enough to build up correlations. An example of such a system with correlations is nuclear matter. In heavy ion collisions, the nuclear matter is first excited and compressed and then decompressed. On a macroscopic scale, the nuclear matter is formed during a supernova collapse and becomes the material of which a neutron star is made.

Here, we address the formation of correlations in a general nonequilibrium situation. The simplest process in nuclear matter is the formation of deuterons; their number is, besides the numbers of other composite particles (tritons, helium-3's, α-particles), an important observable of heavy ion collisions. A microscopic approach to treat this complicated process uses the Boltzmann equation. In this context the equation has been numerically solved utilizing the Beth-Uehling-Uhlenbeck approach, see e.g. Ref. [1]. For nucleon (f_N) and deuteron (f_d) Wigner distribution functions, the Boltzmann equation reads

$$\partial_t f_N + \{U, f_N\} \quad = \quad \mathcal{K}_N^{\rm in}[f_N, f_d]\,(1 - f_N) - \mathcal{K}_N^{\rm out}[f_N, f_d]\, f_N,$$

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$$\partial_t f_d + \{U, f_d\} = \mathcal{K}_d^{\rm in}[f_N, f_d]\,(1+f_d) - \mathcal{K}_d^{\rm out}[f_N, f_d]\,f_d, \quad (1)$$

where U is a mean-field potential and $\{\cdot,\cdot\}$ denotes the Poisson brackets. The collision integrals $\mathcal{K}$ appearing in Eq. (1) couple the distribution functions and respect all the collisions (elastic and reactive) between the constituents. We focus on the reaction part only, e.g. the loss reaction for the deuteron is given by

$$\begin{aligned}\mathcal{K}_d^{\rm out}(P,t) &= \int d^3k \int d^3k_1 d^3k_2 d^3k_3\ |\langle k_1k_2k_3|U_0|kP\rangle|^2_{dN\to pnN} \\ &\quad \times \bar f_N(k_1,t)\bar f_N(k_2,t)\bar f_N(k_3,t) f_N(k,t) \\ &\quad + \ldots \end{aligned} \quad (2)$$

where $\bar f = 1 \pm f$ for bosons/fermions. The transition matrix element is for the break-up operator U_0 and given by the solution of the Alt-Grassberger-Sandhas (AGS) equation [2] explained below. Several approximations have been used to describe the transition matrix U_0, e.g. Born approximation that may be justified in the context of weak interactions, or impulse approximation that is justified for higher scattering energies. One further strategy is to introduce the cross section in the above equations and use experimental values or meaningful extrapolations (assuming detailed balance for the back reaction).

2 In-medium three-body reactions

As nuclear matter provides a dense system we address the question to what extend the reaction *depends on the embedding medium.* To this end we have derived an AGS-type equation for the three-particle correlation embedded in an uncorrelated medium [4,5,6]. This will be shortly sketched here. The equation is derived in the context of the cluster expansion or Dyson equation approach [7]. The expansion is consistent in the sense that the respective in-medium one- and two-body problems are solved and implemented in the three-body equation, i.e. nucleon self-energies ε, Pauli blocking factors, and the Mott effect of the deuteron are all consistently included. The c.m. momentum is treated using four-body kinematics. Note, that the c.m. momentum enters only parametrically into the three-body equation. The three-body Green function evaluated in an uncorrelated medium is then given by

$$\begin{aligned}G_3(z) &= \frac{\bar f_1 \bar f_2 \bar f_3 + f_1 f_2 f_3}{z - \varepsilon_1 - \varepsilon_2 - \varepsilon_3} \\ &\quad + \frac{(\bar f_1 \bar f_2 - f_1 f_2) V_2(12) + \text{perm.}}{z - \varepsilon_1 - \varepsilon_2 - \varepsilon_3}\, G_3(z), \end{aligned} \quad (3)$$

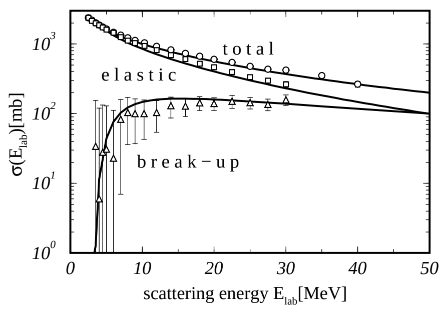

Figure 1: Neutron deuteron cross sections. Experimental data Schwarz et al. [3]

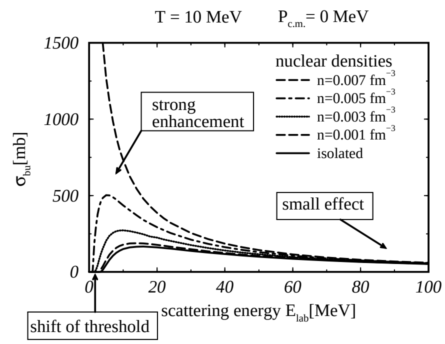

Figure 2: Break-up cross section for different densities at a temperature $T = 10$ MeV.

where the corresponding single particle self-energy in Hartree-Fock approximation is given by

$$\varepsilon_1 = \frac{k_1^2}{2m_1} + \Sigma^{HF}(1), \qquad \Sigma^{HF}(1) = \sum_2 \left[V_2(12,12) - V_2(12,21)\right] f_2, \tag{4}$$

We assume the dominant two-body interaction V_2 only. For a recent and throughout investigation of nuclear three-body forces see Ref. [8]. A similar equation has been derived for the channel† Green function $G_3^{(\alpha)}(z)$, for the two-body subchannel [6]. The AGS transition operator $U_{\alpha\beta}(z)$ is defined by

$$G_3(z) = \delta_{\alpha\beta} G_3^{(\alpha)}(z) + G_3^{(\alpha)}(z) U_{\alpha\beta}(z) G_3^{(\beta)}(z). \tag{5}$$

For $f \to 0$ this operator leads to the isolated AGS-operator with the correct reduction formulas. The resulting in-medium AGS equation for $U_{\alpha\beta}(z)$ is then

$$\begin{aligned} U_{\alpha\beta}(z) \quad &= \quad (1-\delta_{\alpha\beta}) \left[N_3 R_3^{(0)}\right]^{-1} \\ &\quad + \sum_{\gamma\neq\alpha} N_3^{-1} N_2^{(\gamma)} T_3^{(\gamma)}(z) R_3^{(0)} N_3 \, U_{\gamma\beta}(z). \end{aligned} \tag{6}$$

To simplify notation we have introduced $N_3 = \bar f_1 \bar f_2 \bar f_3 + f_1 f_2 f_3$ and for the two-body subchannel of the three-body system $N_2^{(3)} = \bar f_1 \bar f_2 - f_1 f_2$ (cycl. perm.) as

†The three-body fragmentation channels are labeled by the spectator particle $\alpha \in \{1, 2, 3\}$ and the break-up channel by zero.

well as the three-body resolvent $R_3^{(0)}(z) = (z - \varepsilon_1 - \varepsilon_2 - \varepsilon_3)^{-1}$. The respective two-body t matrix in the subchannel γ entering in the above equation is

$$T_3^{(\gamma)}(z) \quad = \quad V_2^{(\gamma)} + V_2^{(\gamma)} R_3^{(0)}(z) N_2^{(\gamma)} T_3^{(\gamma)}(z). \tag{7}$$

The optical theorem has been used to calculate the break-up, the total and the elastic cross sections. For the isolated three-body case the respective theoretical results are shown in Fig. 1 and compared to the latest experimental data on neutron deuteron scattering. The calculation is done using a rank one Yamaguchi potential for the $^1\mathrm{S}_0$ and coupled $^3\mathrm{S}_1 - ^3\mathrm{D}_1$ channels. The effect of the nuclear medium is seen in Fig. 2. The specific effects on the cross section are indicated in the figure. Note that the threshold shifts to smaller energies as it should, since the deuteron binding energy becomes smaller with increasing density. As the binding energy of the two-body subsystem approaches zero one might expect an infinite number of states in the corresponding three-body bound state (Efimov effect). Note, however, that in the three-body system the binding energy of the subsystem is not a fixed quantity but depends on the c.m. momentum of the subsystem, i.e. the binding energy varies in the three-body bound state and therefore new bound states are not expected to appear.

3 Reaction time scales

One consequence of this strongly enhanced cross section can be directly noticed in the time scales involved. To this end we linearize the Boltzmann equation given in (1) as e.g. done in Ref.[5] in the context of the Green function approach. Linearizing with respect to small variations of the deuteron distribution $\delta f = f^0 - f$ from the equilibrium one f^0 leads to

$$\frac{\partial}{\partial t}\,\delta f_d(P,t) = -\frac{1}{\tau_{\mathrm{bu}}[P,n,T]}\,\delta f_d(P,t), \tag{8}$$

where we have introduced the deuteron break-up time $\tau_{\mathrm{bu}}[P,n,T]$ that depends on the density n, temperature T, and deuteron momentum P. Using the cross section and the relative velocities the deuteron break-up time may be written as

$$\tau_{\mathrm{bu}}^{-1} = \frac{4}{(2\pi)^3}\int d^3k_N\,|\boldsymbol{v}_d - \boldsymbol{v}_N|\,\sigma_{\mathrm{bu}}(P)\,f_N^0(\varepsilon). \tag{9}$$

The numerical result is shown in Fig. 3, where we compare the use of the isolated to the in-medium break-up cross section in Eq. (9) at $T = 10$ MeV and a nuclear density of $n = 0.007\ \mathrm{fm}^{-3} \simeq n_0/25$, n_0 normal nuclear matter

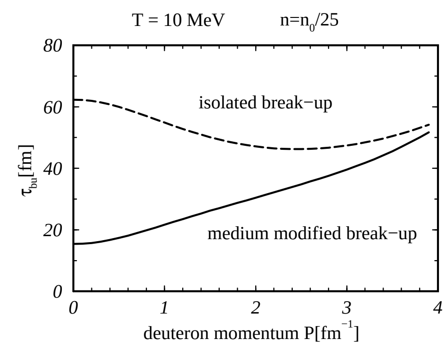

Figure 3: Deuteron break-up time as a function of the deuteron momentum.

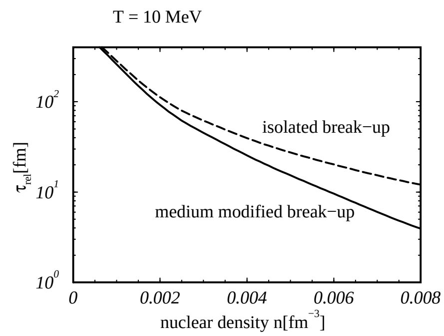

Figure 4: Chemical relaxation time at $T = 10$ MeV as a function of nuclear density.

density. The medium dependent elementary cross section leads to shorter fluctuation times. As expected the difference is vanishing for larger momenta.

A similar result is found for the relaxation time to chemical equilibrium as explained in the following. Linearizing the corresponding rate equations

$$\begin{aligned}\frac{d}{dt}n_d(t) &= -\alpha(t)n_N(t)n_d(t) + \beta(t)n_N^3(t)\\ \frac{d}{dt}n_N(t) &= 2\alpha(t)n_N(t)n_d(t) - 2\beta(t)n_N^3(t)\end{aligned}$$

with respect to small distortions $\delta n = n^0 - n$ from the equilibrium distribution n^0, viz.

$$\frac{d}{dt}\,\delta n_d(t) = -\frac{1}{\tau_{\rm rel}[n,T]}\,\delta n_d(t), \tag{10}$$

leads to the corresponding relaxation time (detailed balance provided)

$$\tau_{\rm rel}^{-1} = \int d^3P d^3k\;|\boldsymbol{v}_N - \boldsymbol{v}_d|\sigma_{\rm bu}(P)\frac{f_N(k,t)}{n_N(t)}\frac{f_d(P,t)}{n_d(t)}\;\,[n_N + 4n_d]\,. \tag{11}$$

In Fig. 4 we show the comparison of using the different cross sections under discussion. Again the difference in time scales is significant, increasing with increasing density [10].

Another well known many-body feature of bound states is the Mott effect. For fermions bound states should vanish beyond a certain density. This density is momentum dependent (Mott momentum) that is shown in Fig. 5. Below the lines shown no pole exist in the respective few-body equations. Using

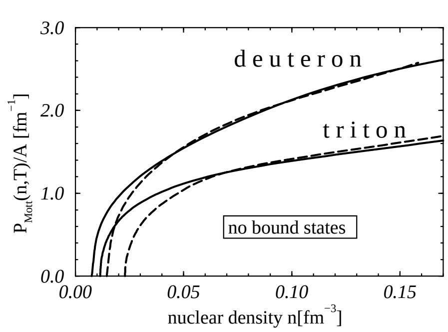

Figure 5: Mott momentum for deuteron and tritons at $T = 10$ MeV (solid lines) and $T = 20$ MeV (dashed lines).

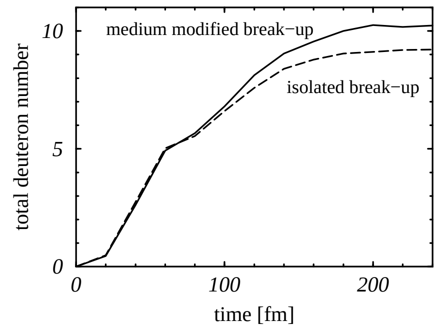

Figure 6: Total number of deuterons in time. Due to statistics and temperature the lines are considered preliminary.

the present approach this is elaborated in detail for the three-body system in Ref. [11]. The Mott effect is an important ingredient in microscopic simulations of heavy ion collisions, see e.g. [1], and may have consequences for the triton to helium-3 ratio in asymmetric nuclear matter.

4 Extension to heavy ion collisions

So far the investigation has been restricted to nuclear matter (homogeneous and symmetric). A laboratory situation closest to nuclear matter is provided by heavy ion collisions. As a test case we investigate the collision $^{129}\mathrm{Xe}+^{119}\mathrm{Sn}$ at 50 MeV/A lab. energy. This collision has been investigated by the INDRA collaboration, see e.g. Ref. [12].

The density profile suggests that the medium effects influence the deuteron formation in the final stage of the heavy ion collision, because of the Mott effect. The temperature in this final stage extracted from the BUU simulation assuming a Fermi distribution is rather homogeneous and $T \simeq 4 \ldots 6$ MeV. This is compatible with results we achieved with the Quantum Molecular Dynamics code provided by Aichelin [13] and other calculations [14]. Fig. 6 refers to a preliminary calculation using the cross sections shown in Fig. 2 that have been implemented in the BUU code [1]. The number of deuterons is increasing up to 10%. Also, note that the in-medium effect becomes visible at the final stage only. The density is small enough for deuterons to significantly survive.

Finally, we address the question of chemical equilibrium during the heavy ion collision. Fig. 7 shows the total numbers of deuterons and nucleons as given in the BUU simulation (solid line) compared to the law of mass action

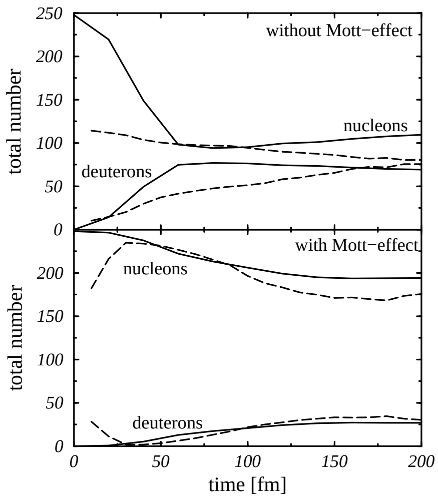

Figure 7: Number of nucleons and deuterons in the BUU simulation (solid lines) compared to the respective law of mass action (dashed lines).

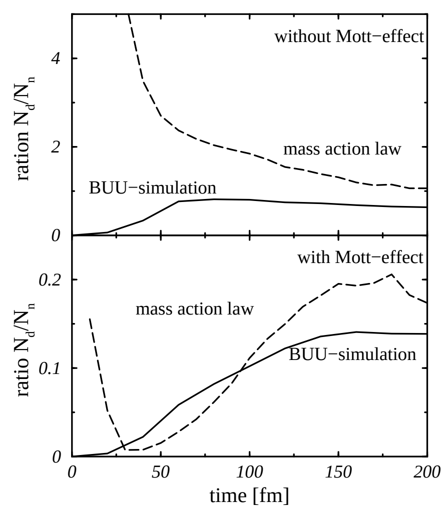

Figure 8: Deuteron to nucleon ratio for the BUU simulation (solid line) compared to the law of mass action (dashed line).

result. The upper figure ignores the Mott effect (that would lead to a wrong description of the experimental data), whereas the lower figure shows the same with the Mott effect properly taken into account. The calculation is done without medium modifications for simplicity. Including the Mott effect reduces the number of final deuterons by about a factor of 3 and in turn increases the number of nucleons by about a factor of two. In the deuteron to nucleon ratio this ends up to about a factor of 6 shown in Fig. 8 for the same process. It seems that for the process considered ($^{129}\mathrm{Xe} + ^{119}\mathrm{Sn}$ at 50 MeV/A lab. energy central collision) the evolution is rather close to chemical equilibrium provided the Mott effects are properly accounted for.

5 Conclusion and Outlook

In summary, we have presented a systematic and consistent decoupling scheme to handle the many-body problem. This leads in turn to in-medium few-body equations that have been rigorously solved for the three-body system to treat the deuteron formation via three-particle collisions. The prominent changes induced by the medium are the *self-energy shifts*, the *Pauli blocking*, the *Mott effect* and the change of *reaction rates*. Within linear response we have shown

that the change in the rates leads also to *faster time scales*. The deuteron production for the BUU simulation of the ^{129}Xe $+^{119}$ Sn at 50 MeV/A lab. energy central heavy ion collision is *enhanced by 10%*.

The exact treatment of few-body systems embedded in the medium is however not restricted to nuclear physics. Further potential applications are possible, e.g., in the field of semiconductors to treat the formation of excitons or the trion bound state in the dense low dimensional electron plasmas. For recent experiments see e.g. [15]. For highly ionized dense plasmas the impulse approximation may fail for the three-particle break-up cross section needed to calculate the ionisation and recombination rates.

Finally, the formalism is capable to also treat the four-particle problem. In this context the α-particle is the most interesting object. Because it is light and has a large binding energy per nucleon, it should play a dominant role in the decompressing hot nuclear matter.

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